Section 10.5
Volume

Objectives

1. Use volume formulas to compute the volumes of three-dimensional figures and solve applied problems.

2. Compute the surface area of a three-dimensional figure.
Formulas for Volume

- Volume of a rectangular solid, $V$, is the product of its length, $l$, its width, $w$, and its height, $h$:
  $$V = lwh$$

- **Volumes of Boxlike Shapes**
  
  - **Rectangular solid**
    
    - $V = lwh$
  
  - **Cube**
    
    - $V = s^3$
Example 1
Solving a Volume Problem

• You are about to begin work on a swimming pool in your yard. The first step is to have a hole dug that is 90 feet long, 60 feet wide, and 6 feet deep.

• You will use a truck that can carry 10 cubic yards of dirt and charges $12 per load. How much will it cost you to have all the dirt hauled away?

• Solution: Begin by converting feet to yards:

\[
90 \text{ ft} = \frac{90 \text{ ft}}{1} \cdot \frac{1 \text{ yd}}{3 \text{ ft}} = \frac{90}{3} \text{ yd} = 30 \text{ yd}
\]

Similarly, \(60 \text{ ft} = 20 \text{ yd}\) and \(6 \text{ ft} = 2 \text{ yd}\).
Example 1 continued

Next, we find the volume of dirt that needs to be dug out and hauled off.

\[ V = lwh = 30 \text{ yd} \cdot 20 \text{ yd} \cdot 2 \text{ yd} = 1200 \text{ yd}^3 \]

Now, find the number of truckloads by dividing the number of cubic yards of dirt by 10 yards.

\[
\text{Number of truckloads} = \frac{1200\text{yd}^3}{10\text{yd}^3/\text{trip}} = \frac{1200\text{yd}^3}{1}/\frac{10\text{yd}^3}{\text{trip}} = \frac{1200}{10} = 120 \text{ trips}
\]

The truck charges $12 per trip, the cost to have all the dirt hauled away is:

\[ 120 \text{ trips} \cdot $12 = 120($12) = $1440 \]
Example 2
Volume of a Pyramid

- The Transamerica Tower in San Francisco is a pyramid with a square base. It is 256 meters tall and each side of the square base is 52 meters long. Find its volume.

- Solution: The area of the square base is:

\[ B = 52\text{m} \cdot 52\text{ m} = 2704\text{ m}^2 \]

The volume of the pyramid is:

\[ V = \frac{1}{3}Bh = \frac{1}{3} \cdot 2704\text{ m}^2 \cdot 256\text{m} \approx 230,742\text{ m}^3 \]
Example 3
Volume of a Right Circular Cylinder

• Find the volume of this cylinder with diameter = 20 yards and height = 9 yards.

• Solution.
The radius is \( \frac{1}{2} \) the diameter = 10 yards

\[
V = \pi r^2 h = \pi (10 \text{ yd})^2 \cdot 9 \text{ yd} \\
= 900\pi \text{ yd}^3 \approx 2827 \text{ yd}^3
\]
Volumes of a Cone and a Sphere

- The Volume, $V$ of a right circular cone that has height $h$ and radius $r$ is given by the formula:

$$V = \frac{1}{3} \pi r^2 h$$

- The Volume, $V$ of a sphere of radius $r$ is given by the formula:

$$V = \frac{4}{3} \pi r^3$$
Example 4
Volumes of a Sphere and Cone

• An ice cream cone is 5 inches deep and has a radius of 1 inch. A spherical scoop of ice cream also has a radius of 1 inch. If the ice cream melts into the cone, will it overflow?

• Solution: The ice cream will overflow if the volume of the ice cream, a sphere, is greater than the volume of the cone. Find the volume of each.
Example 4 continued

\[ V_{\text{cone}} = \frac{1}{3} \pi r^2 h \]
\[ = \frac{1}{3} \pi (1 \text{ in.})^2 \cdot 5 \text{ in.} \]
\[ = \frac{5\pi}{3} \text{ in.}^3 \approx 5 \text{ in.}^3 \]

\[ V_{\text{sphere}} = \frac{4\pi r^3}{3} \]
\[ = \frac{4\pi (1 \text{ in.})^3}{3} \]
\[ = \frac{4\pi}{3} \text{ in.}^3 \approx 4 \text{ in.}^3 \]

The volume of the spherical scoop of ice cream is less than the volume of the cone so there will be no overflow.
Surface Area

- The area of the outer surface of a three-dimensional object.
- Measured in square units
Example 5
Finding the Surface Area of a Solid

• Find the surface area of this rectangular solid.

• Solution:
The length is 8 yards, the width is 5 yards, and the height is 3 yards. Thus, \( l = 8, w = 5, h = 3 \).

\[
SA = 2lw + 2lh + 2wh \\
= 2 \cdot 8 \text{ yd} \cdot 5 \text{ yd} + 2 \cdot 8 \text{ yd} \cdot 3 \text{ yd} + 2 \cdot 5 \text{ yd} \cdot 3 \text{ yd} \\
= 80 \text{ yd}^2 + 48 \text{ yd}^2 + 30 \text{ yd}^2 = 158 \text{ yd}^2
\]